

“ The five practices help me make sure that I know the key points that I’m going to use as the foundation throughout the teaching that occurs from that point forward—so, knowing exactly why I’m doing this task at this point in my sequence of lessons, and exactly what points are like the big rocks and big understandings. ”

—JENNIFER MOSSOTTI, EIGHTH-GRADE TEACHER

CHAPTER 2

Setting Goals and Selecting Tasks



Before you can begin to work on the five practices, you must first set a goal for student learning and select a task that aligns with your goal and providing opportunities for students to learn important mathematics content and to engage in essential mathematical practices. Smith and Stein (2018) have described this as *Practice 0*—a necessary step in which teachers must engage as they begin to plan a lesson that will feature a whole class discussion. As they explain:

To have a productive mathematical discussion, teachers must first establish a clear and specific goal with respect to the mathematics to be learned and then select a high-level mathematical task. This is not to say that all tasks that are selected and used in the classroom must be high level, but rather that productive discussions that highlight key mathematical ideas are unlikely to occur if the task on which students are working requires limited thinking and reasoning. (Smith & Stein, 2018, p. 27)

In this chapter, we first unpack what is involved in setting goals and selecting tasks and illustrate what this practice looks like in an authentic middle school classroom. We then explore challenges that teachers face in engaging in this practice and provide an opportunity for you to explore setting a goal and selecting a task in your own teaching practice.



Part One: Unpacking the Practice: Setting Goals and Selecting Tasks

What does it take to engage in this practice? This practice requires first specifying the learning goal for the lesson and then identifying a high-level task that aligns with the learning goal. The key questions, shown in Figure 2.1, are intended to help you focus on important aspects of the practice.

Figure 2.1 • Key questions that support the practice of setting a goal and selecting a task

WHAT IT TAKES	KEY QUESTIONS
Specifying the learning goal	Does the goal focus on what students will learn about mathematics (as opposed to what they will do)?
Identifying a high-level task that aligns with the goal	Does your task provide students with the opportunity to think, reason, and problem solve?
	What resources will you provide students to ensure that all students can access the task?
	What will you take as evidence that students have met the goal through their work on this task?

In the sections that follow, we provide an illustration of this practice drawing on a lesson taught by Jennifer Mossotti in her eighth-grade classroom. As you read the description of what Mrs. Mossotti thinks about and articulates while planning her lesson, consider how her attention to the key questions influence her planning.

Specifying the Learning Goal

Your first step in planning a lesson is specifying the goal(s). Consider Goals A and B for each of the mathematical ideas targeted in Figure 2.2. How are the goals the same and how are they different? Do you think the differences matter?

For each of the mathematical ideas targeted in Figure 2.2, the goal listed in Column A is considered a performance goal. Performance goals indicate what students will be able to do as a result of engaging in the lesson. By contrast, each of the goals listed in Column B is a learning goal. The learning goals explicitly state what students will understand about mathematics as a result of engaging in a particular lesson. The learning goal needs to be stated with sufficient specificity such that it can guide your decision-making during the lesson (e.g., what task to select for students to work on, what questions to ask students as they work



Figure 2.2 • Different goals for learning specific mathematical ideas

TARGETED IDEA	GOAL A	GOAL B
Slope	Students will be able to find the slope of a line given two points.	Students will recognize that slope is the ratio of the vertical change to the horizontal change between <i>any</i> two points on the line.
Multiplying binomials	Students will be able to use FOIL (first-outer-inner-last) to find the product of any two binomials.	Students will recognize that the binomials $(x + a)$ and $(x + b)$ are factors, each of which can represent a dimension of a rectangle, and that the product of these factors is an area. The resulting area can be represented algebraically as $x^2 + ax + bx + ab$. This algebraic representation also results from applying the distributive property twice: $(x + a)(x + b) = x(x + b) + a(x + b) = x^2 + bx + ax + ab$.
Proportions	Students will be able to use cross multiplication to find the missing value in problems where the quantities being compared are in a proportional relationship.	Students will recognize that a proportion consists of two ratios that are equivalent to each other (e.g., $\frac{a}{b} = \frac{ax}{bx}$) and that missing values in the proportion can be found by determining the scale factor x that relates the two ratios or by determining the relationship (multiplicative) between a and b and recognizing that ax and bx must have the same relationship.

on the task, which solutions to have presented during the whole class discussion). According to Hiebert and his colleagues (2007):

Without explicit learning goals, it is difficult to know what counts as evidence of students' learning, how students' learning can be linked to particular instructional activities, and how to revise instruction to facilitate students' learning more effectively. Formulating clear, explicit learning goals sets the stage for everything else. (p. 51)

In general, “the better the goals, the better our instructional decisions can be, and the greater the opportunity for improved student learning” (Mills, 2014, p. 2).

According to Hunt and Stein (in press), “too often, we define what mathematics we wish students to come to ‘know’ as performance, or what students will ‘do,’ absent the understandings that underlay their behaviors.” If we want students to learn mathematics with understanding, we need to specify what exactly it is we expect them to understand about mathematics as a result of engaging in a lesson. Hence, goals you set for a lesson should focus on what is to be learned not solely on performance.

Mrs. Mossotti was beginning a unit on linear functions with her eighth-grade students. Since students’ experience with linear functions in



seventh grade had been limited to proportional relationships, for this lesson she wanted students to be able to find the rate of change and interpret the y -intercept in a context where the relationship between variables was not proportional. As a result of the lesson, she wanted her students to understand that:

1. The rate of change can be seen as the ratio of the change in the y -variable compared to the change in the x -variable, as the rate expressed with the words “for each, per, for every” in a verbal description or as the coefficient of x in the equation $y = mx + b$.
2. Some functions do not *start* at zero. That is, the point $(0,0)$ is not a solution for all linear functions.
3. The y -intercept can be understood as the initial value of a linear function in a real-world context.

TEACHING TAKEAWAY

Specificity is one of the keys to setting learning goals and finding appropriately aligned tasks.

This level of specificity at which Mrs. Mossotti articulated the learning goals for the lessons will help her in identifying an appropriate task for her students, and subsequently, it will help her in asking questions that will move students toward the goal and in determining the extent to which students have learned what was intended.

Identifying a High-Level Task That Aligns With the Goal

Your next step in planning a lesson is to select a high-level task that aligns with the learning goal. High-level or cognitively challenging mathematical tasks engage students in reasoning and problem solving and are essential in supporting students’ learning mathematics with understanding. By contrast, low-level tasks—tasks that can be solved by applying previously learned rules and procedures—require limited thinking or understanding of the underlying mathematical concepts. According to Boston and Wilhelm (2015, p. 24), “if opportunities for high-level thinking and reasoning are not embedded in instructional tasks, these opportunities rarely materialize during mathematics lessons.” In addition, research provides evidence that students who have the opportunity to engage in high-level tasks on a regular basis show greater learning gains than students who engage primarily in low-level tasks during instruction (e.g., Stein & Lane, 1996; Stigler & Hiebert, 2004; and Boaler & Staples, 2008).

Tasks that provide the richest basis for productive discussions have been referred to as *doing-mathematics* tasks. Such tasks are nonalgorithmic—no solution path is suggested or implied by the task and students cannot solve them by the simple application of a known rule. Hence students must explore the task to determine what it is asking them to do and develop and implement a plan drawing on prior knowledge and experience to solve the task (Smith & Stein, 1998).

While the level of cognitive demand is a critical consideration in selecting a task worthy of discussion, there are other characteristics that you should also consider when selecting a task. These characteristics help ensure that students will have the opportunity to engage in the mathematics practices/processes (e.g., make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments) that are viewed as essential to developing mathematical proficiency. When sizing up the potential of a task, keep in mind the questions shown in Figure 2.3.

Figure 2.3 • Questions to help you size up the potential of a task

- Are there multiple ways to enter the task and to show competence?
- Does the task require students to provide a justification or explanation?
- Does the task provide the opportunity to use and make connections between different representations of a mathematical idea?
- Does the task provide the opportunity to look for patterns, make conjectures, and/or form generalizations?

Selecting tasks that have these additional characteristics does not guarantee that students will engage in the mathematical practices. However, the use of the five practices together with such tasks will help ensure that this will occur. So rather than thinking about separate process goals, such as the Standards for Mathematical Practice advocated for in the Common Core State Standards for Mathematics (2010), we encourage you to consider characteristics of tasks that will provide your students with the opportunities to engage in such processes.

For her lesson, Mrs. Mossotti created the State Fair Task, shown in Figure 2.4. She selected the context of the state fair for three reasons. First, the context was relatable to her students since the New York State Fair took place in her students' hometown. Second, the y -intercept had meaning in this context since attendees have to pay a fee to enter the fair and ride tickets are an additional cost that you pay after entry. Third, she thought that there would be different ways to enter and solve the task.

Mrs. Mossotti carefully chose the points that she would include on the graph. As she explained:

I purposely chose the values here of one ticket purchased, plus the entry fee, eight tickets and ten tickets. If I had chosen two points of consecutive amounts of tickets purchased, I think it would have been too obvious for them to figure out the ticket price. [So] I chose eight tickets purchased and ten tickets purchased for a difference of two

TEACHING TAKEAWAY

Doing-mathematics tasks, rather than procedural exercises, lend themselves to rich and productive mathematical discussion.




tickets, and if they can understand the difference of two tickets will have a different cost, then they can start to work from there. But at the same time, I've also purposely put in the cost for one ticket plus the entry fee. Some of them [may] think that this is a price per ticket. I'll ask questions so that they understand that if this is the price for one ticket, how come eight of them are only just a little bit more?

In order to ensure that students would have access to the task, Mrs. Mossotti planned to provide students with resources on which they could draw in solving the task—but would leave it up to the students to

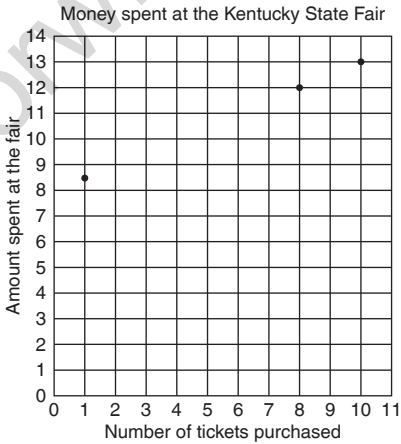
Figure 2.4 • The State Fair task

The State Fair

You are going to the Kentucky State Fair in August. You are trying to figure out how much you should plan to spend. The graph below shows how much three different people spent after going through the main gate and then buying their ride tickets. Every ride ticket is the same price.



Money spent at the Kentucky State Fair



Number of tickets purchased	Amount spent at the fair
1	8.5
8	12
10	13

1. After entering the fair, you decide to buy four ride tickets. What will be your total cost for attending the fair? How do you know?
2. Describe how the cost increases as you buy more tickets. Be specific.
3. After entering the fair, you decide you want to go on a lot of rides. What will be the total cost for attending the fair and then purchasing 15 ride tickets?
4. Write a description, in words or numbers and symbols, that can be used to find the total cost after entering the fair and purchasing any number of tickets.
5. How does the ticket price appear in your description or expressions?
6. How does the ticket price appear in the graph?

Extension

1. If you went to the Kentucky State Fair, how many ride tickets could you buy with \$25.00?
2. If you could enter the Kentucky State Fair for free, how would the graph look different?

Source: Jennifer Mossotti. Ferris wheel photo by Hannah Morgan on Unsplash.

decide which, if any of them, would be useful. These included: rulers (so students could draw a line connecting the three points), calculators (so students could compute quickly and accurately), patty paper (so students could trace the vertical and horizontal change between two points and move it like a transformation), extra blank paper (so students had plenty of space to try things out), and highlighters (so students could make some aspect of the graph salient). In addition to these material resources, Mrs. Mossotti also decided that she would provide *human* resources by having students work on the task in pairs or trios so that they would have others with whom to confer.

When asked what students would say, do, or produce that would provide evidence of their understandings of the goals in the lesson through their work on this task, Mrs. Mossotti was quite specific. She indicated that students would do and say things such as:

- It costs \$1.00 more for two more tickets so it must be 50¢ for each ticket (*determined the rate of change*).
- The cost for 0 tickets is NOT \$0.00 (*recognized that the y-intercept is not 0*)
- I can add 50¢ three more times to \$8.50 to find the total cost for four tickets (*determined the rate of change*).
- The increase in price from one ticket to eight tickets is \$3.50, so if I divide \$3.50 by 7, I get 50¢. So that must be the cost per ticket (*determined the rate of change*).
- The cost for every/for each/per/for one ticket is 50¢ (*determined the rate of change*).
- Since the graph does not start at zero, I cannot just divide the total amount spent by the number of tickets to find the price per ticket (*recognized that the y-intercept is not 0 so that the relationship is not proportional*).
- No matter how many tickets I decide to buy, I have to pay money to enter the fair before buying the ride tickets (*recognized that the y-intercept has meaning in the context*).

Jennifer Mossotti's Attention to Key Questions: Setting Goals and Selecting Tasks

During her initial stage of lesson planning, Mrs. Mossotti paid careful attention to the key questions. First, in setting her goals for the lesson, she clearly articulated what it was she wanted students to learn about mathematics as a result of engaging in the task. The specificity with which she stated her goals made it possible to determine what students understood about these ideas and to formulate questions that would help



move her students forward. While she wanted her students to determine the rate of change, she also wanted to make sure that they understand what rate of change is and how it is represented graphically.

Second, she selected a high-level *doing-mathematics* task that aligned with her goals. Students could not solve the state fair task by application of a known rule or procedure; it would require students' perseverance and sense making. She *sized up* the task (see Figure 2.3) to ensure that it had several other important characteristics. Specifically, there were a number of general approaches that students could use to enter the task (e.g., making a table, drawing a line connecting the three points, focusing on the change between two points, tracing the points/line and translating it) and the material and human resources that the teacher planned to provide would support their work. The task used several different representations—it began with both a context and a graph and asked students to write a description using words or numbers and symbols—and it required students to explain, "How do you know?" (Question 1). Finally the task asked students to generalize their findings by describing in words or numbers and symbols how to find the cost of entering the fair and purchasing any number of tickets. Hence through their work on the task, students could learn important mathematics and engage in key practices.

The State Fair task, along with the resources Mrs. Mossotti made available to students, allowed all students to enter the task at some level. Rather than differentiating instruction by providing different students with different tasks, she selected one task and met the needs of different learners by providing a range of resources for students to consider and questions that would challenge learners at different levels. For example, the inclusion of the extension questions provided a challenge for students who were able to quickly make progress on the task, while questions 1–3 provided scaffolding for writing the general description in question 4 for other students.

Finally, Mrs. Mossotti indicated some things that she expected students to say and do that would provide evidence students were making progress on the ideas she wanted them to learn. By considering this evidence in advance of the lesson, she was ready to pay close attention to students' work for indications that they were making progress in their understanding.

Through her careful attention to setting goals and selecting a task, Mrs. Mossotti's planning was off to a productive start and she was ready to engage in the five practices. In the next chapter we will continue to investigate her planning process as she anticipates what she thinks her students will do when presented with the task and how she will respond. We now turn our attention to the challenges that teachers face in setting goals and selecting tasks.

Part Two: Challenges Teachers Face: Setting Goals and Selecting Tasks

As we described in the chapter opening, setting goals and selecting tasks are foundational to orchestrating productive discussions. Setting goals and selecting tasks, however, are not without their challenges. In this section, we focus on four specific challenges associated with this practice, shown in Figure 2.5, that we have identified from our work with teachers.

Figure 2.5 • Challenges associated with setting goals and selecting tasks

CHALLENGE	DESCRIPTION
Identifying learning goals	Goal needs to focus on what students will learn as a result of engaging in the task, not on what students will do. Clarity on goals sets the stage for everything else!
Identifying a doing-mathematics task	While <i>doing-mathematics</i> tasks provide the greatest opportunities for student learning, they are not readily available in some textbooks. Teachers may need to adapt an existing task, find a task in another resource, or create a task.
Ensuring alignment between task and goals	Even with learning goals specified, teachers may select a task that does not allow students to make progress on those particular goals.
Launching a task to ensure student access	Teachers need to provide access to the context and the mathematics in the launch but not so much that the mathematical demands are reduced and key ideas are <i>given away</i> .

Identifying Learning Goals

Identifying learning goals is a challenging but critical first step in planning any lesson. It is challenging because we often focus on what students are going to be able to do as a result of engaging in a lesson, not on what they are going to learn about mathematics. When Michelle Saroney, one of our three featured teachers, first described her goal for the lesson she planned to teach in her sixth-grade classroom, she stated her goal in terms of the standard she wanted to address: “interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions” (NY6.NS – New York State Next Generation Mathematics Learning Standards, p. 79). When pressed to be more specific about what she wanted students to learn about mathematics, she acknowledged, “The goals are my most difficult part of planning tasks.” While Mrs. Saroney was clear regarding what her students would do—solve the pizza party task shown in Figure 2.6—she struggled to articulate what she wanted

them to understand about mathematics as a result of engaging in the task. [NOTE: This standard is one of the most misunderstood standards at Grade 6 and often translates into instruction on the traditional algorithm with no visual models. See <https://achievethecore.org/aligned/misunderstood-middle-school-mathematics-standards-grade-6/>]


Figure 2.6 • The Pizza Party task

Pizza Party

You ordered pizza for your birthday party. When the party was over you still had $4\frac{5}{6}$ pizzas left over. Your mother decided to freeze the remaining pizza. She put $\frac{2}{3}$ of a pizza (one serving) in each freezer bag.

1. How many servings would your mother be able to freeze?
2. How much more pizza does your mother need to make another serving?

Draw a picture, build a model, construct a number line, or make a table to explain your solution.



Source: Task adapted from Nolan, Dixon, Roy & Andreasen, 2016.

Image Source: bonetta/iStock.com

During the discussion with her colleagues regarding her goals and task, Mrs. Saroney indicated that this would be the first time that students would be solving a division problem that involved a mixed number and the first time this year that they would be solving a contextual problem. Through the conversation she tried to clarify in detail what understandings of mathematics students would need to solve the pizza task and as a result determined the following learning goals for the lesson. Specifically, as a result of engaging in the lesson, she wanted her students to understand that:

1. When you scale a fraction up or down, you have not changed the amount it represents ($\frac{2}{3} = \frac{4}{6}$); equivalent fractions represent the same area and name the same position on a number line. *Since the mixed number and the fraction in the task did not have the same denominator, students would need to be able to rewrite $\frac{2}{3}$ as $\frac{4}{6}$ and know that they were equivalent.*
2. When you are dividing by a fraction, the remainder is expressed as a fraction of the divisor. *The $\frac{1}{6}$ of a pizza left over after making 7 servings needs to be interpreted as $\frac{1}{4}$ of a serving.*
3. When you find “how many ___ are in ___?” you are doing division. That is, in $a \div b$ you are trying to find how many times b is contained in a . *What division actually means whether you are working with fractions or whole numbers.*

Mrs. Saroney also noted that the task had many features that would provide opportunities for her students to engage in the mathematical

practices, including asking for an explanation and using and making connections between different representations.


With new clarity regarding what she wanted students to understand, Mrs. Saroney was now ready to anticipate what students would do with the task and prepare questions that would help her illuminate what her students understood about these ideas.

Mrs. Saroney is not alone in her struggle to identify learning goals. Consider, for example, a lesson that Neil Tanner was planning for his eighth-grade students featuring the downloading music task (see Figure 2.7). Mr. Tanner initially indicated that his goal for the lesson was “for students to find the point of intersection of two linear functions.” This goal clearly stated what students were going to do but provided no insight into what he wanted his students to understand about systems of equations. Here are a few things Mr. Tanner *could* target explicitly in his lesson:

- The solution to a system of two unique nonparallel linear equations in two variables is represented graphically by the point of intersection of the lines, and it is represented by the ordered pair (x, y) that makes both equations true statements or satisfies the equations simultaneously.
- Two unique nonparallel linear equations switch positions at the point of intersection—the equation with the y -intercept closer to $(0, 0)$ and the larger rate of change will be closer to the x -axis before the point of intersection and the equation with the y -intercept furthest away from $(0, 0)$ and a smaller rate of change will be closer to the x -axis after the point of intersection.
- Systems of linear equations can be solved using tables, graphs, and equations and the different representations can be connected.

Figure 2.7 • Downloading Music task

Downloading Music



You are trying to decide which service you should use to download music. TUNE IN charges \$1.00 for each song you download, plus an \$8.00-per-month membership charge. NOTEABLE charges 50¢ for each song you download, plus a \$12.00-per-month membership charge. How many songs would you have to download in a month before NOTEABLE is a better deal? Explain your reasoning.

Image Source: oleksii arseniuk/iStock.com

Why does this level of specificity matter? It matters because with this level of specificity, Mr. Tanner will be able to determine not only whether or not his students can find the solution to the system, but



whether they can explain what the solution means (first bullet), begin to see how systems of equations work in general (second bullet), and see that different representations can be used and connected to each other (third bullet). So when Mr. Tanner actually interacts with his students as they work on the task, with these targets in mind, he can press them to explain what happens at, before, and after the point of intersection and how the rate of change and the y -intercept impact the relative position of the function. In addition, this level of specificity will help Mr. Tanner consider the solutions he would want students to share during the whole class discussion and the questions he wants to ask about them later in the lesson. Hence, the specificity of the goal is going to provide guidance to him during the lesson and help him determine what his students do and do not understand. This information will then help him in planning subsequent lessons. (See Hunt and Stein (in press) for a description of three interconnect phases that teachers can use individually or collaboratively to create and refine goals for student learning.)

Identifying a Doing-Mathematics Task

While doing-mathematics tasks provide the optimal vehicle for whole-class discussions, not all curricular materials are replete with such tasks. Traditional textbooks tend to feature more procedural tasks that provide limited opportunities for reasoning and problem solving. While such resources do include *word problems*, they are often solved using procedures that have been previously introduced and modeled and require limited thinking. While standards-based texts (Senk & Thompson, 2003) contain some procedural tasks, they also include high-level tasks that promote reasoning and problem solving. If you are using a resource that does not include high-level tasks, what should you do? In this section we will explore three possible options—modify an existing task, find a task in another resource, or create your own task.

Adapting an Existing Task

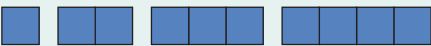

Some textbook tasks give students too much information and leave little for students to figure out on their own. Take for example the Patterns task shown in Figure 2.8. In the original task (left side of Figure 2.8), the table is done, equations are provided, and very little thinking is needed. Part 1 requires determining which equation returns the correct value for P when values for t are substituted. Part 2 is a *plug and chug* task that can be solved by substituting 20 in the equation selected in Part 1. While students are asked to graph in Part 3, this is nothing more than a plotting points exercise.

When Elaine Richard saw the Patterns task in her algebra textbook, she decided to modify the task so that her students would have to do more

TEACHING TAKEAWAY

If your textbook does not provide worthwhile enough tasks for discussion, you have options: adapt, seek out, or create your own!

Figure 2.8 • The original and modified Patterns task

Patterns—Original					Patterns—Modified				
The table of value below describes the perimeter of each figure in the pattern of blue tiles. The perimeter P is a function of the number of tiles t .					The diagram below shows the first four figures in the square pattern. The first figure has one square. For each additional figure, one new square is added.				
t	1	2	3	4					
P	4	6	8	10	1. Compute the perimeter of the first four figures.				
					2. Draw the fifth and sixth figures and compute their perimeters.				
1. Choose a rule to describe the function in the table.					3. Describe in words how to draw the 10th figure and what the perimeter would be.				
A. $P = t + 3$					4. Describe how to find the perimeter of the 100th figure and explain how the description relates to the pattern of tiles.				
B. $P = 4t$					5. Write a rule to relate the perimeter P to the number of tiles t . Explain how each part of your formula relates to the tile pattern.				
C. $P = 2t + 2$					6. Graph the function. Why does it make sense that the graph has the shape it does in relation to the pattern?				
D. $P = 6t - 2$									
2. How many tiles are in the figure if the perimeter is 20?									
3. Graph the function.									

thinking and reasoning. In comparing the original task to the modified task, you will see that Ms. Richard made the following changes:

- The table has been eliminated and instead students must determine the perimeters of the first four trains from the figure.
- Students now need to draw the next two figures and to describe and compute the perimeter of larger figures without drawing and counting.
- Rather than determine which of the given equations produces the table of values, students must write a rule that could be used to find the perimeter of any figure in the pattern and explain how each part of the rule relates to the square pattern.
- In addition to graphing the function, students must explain why the graph of the function is linear.

By making these changes, Ms. Richard has transformed a low-level task into a doing-mathematics task. In addition, her modified task has the additional characteristics we previously discussed—it can be entered and solved in several different ways, it requires students to use and make connections between different representations, it asks students to explain

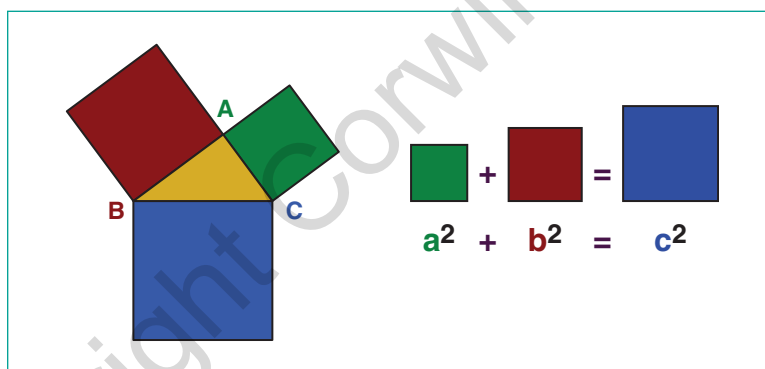


their thinking, and it asks students to generalize. (See Arbaugh, Smith, Boyle, Stylianides, and Steele, 2018; Boyle & Kaiser, 2017; and Smith & Stein, 2018 for more insight on how to modify tasks.)

Finding a Task in Another Resource

You can find high-level doing-mathematics tasks in many print and electronic resources. The challenge is to find a task that meets your mathematical needs, is accessible to your students, and fits with the content and flow of your curriculum. Consider, for example, Devon Washington's experience at the beginning of a unit on the Pythagorean theorem. Although his textbook included pictures that showed why $c^2 = a^2 + b^2$ (e.g., Figure 2.9), there was little for students to figure out before they began to use the formula to find missing values for a , b , and c .

Figure 2.9 • Visual and symbolic representation of the Pythagorean theorem



Source: Adapted from Image by Openclipart-Vectors, pixabay.com CC0 Creative Commons <https://creativecommons.org/publicdomain/zero/1.0/deed.en>

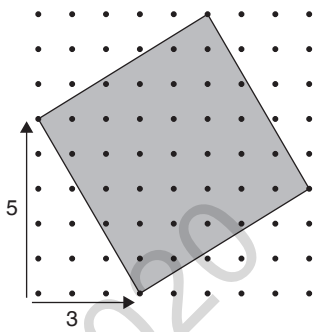
Mr. Washington wanted his students to *discover* the theorem so that they would know not just what it was but where it came from and why it made sense. As a result, he decided to replace the opening activity in his book with one he found online at map.mathshell.org. The lesson, entitled “Discovering the Pythagorean Theorem” (MARS Shell Center Team, 2007–2015), featured a series of activities that provide students the opportunity to find areas of *tilted* squares (see an example in Figure 2.10), to look for patterns by investigating the areas of different squares, to make a conjecture about any x by y square, and to consider visual proofs of the theorem. (The entire lesson can be found at <http://map.mathshell.org/lessons.php?unit=8315&collection=8>.)

Figure 2.10 • The first activity in the “Discovering the Pythagorean Theorem” lesson

The dots on the grid are all one unit apart.

The square shown here can be described as a '3 by 5' square. The first number in the description *always* represents the horizontal tilt of the square.

- Find its area.
Show all your reasoning.



Source: This task is part of a carefully engineered lesson, “Discovering the Pythagorean Theorem” from the Mathematics Assessment Project, which can be downloaded for free from <http://map.mathshell.org/lessons.php?unit=8315&collection=8>

By replacing the first task in the unit with a doing-mathematics task, Mr. Washington provided his students with the opportunity to develop a conceptual understanding of the Pythagorean theorem. This became a base on which he could then build procedural fluency in using the theorem and subsequently for using the theorem to compute the distance between two points and derive the distance formula. (For an extended discussion on building procedural fluency from a base of conceptual understanding, see Smith, Steele, & Raith, 2017.)

Mr. Washington found a website that had a storehouse of good tasks. But not all websites are of equal value. In Appendix A, we have included a list of web resources that may be helpful to you in finding doing-mathematics tasks. This list is not exhaustive but should be helpful in getting started.

Creating a Task

If you cannot find a task you want to use, you may decide to create one yourself. This was the case for Nancy Haines. Ms. Haines’s eighth-grade students were beginning a unit on exponential functions. While the unit in her book actually started with a good task—folding a sheet of paper into equal parts and counting the number of regions formed with each fold—she wanted to start the unit with a task that she thought students would find more engaging. Toward this end, she created the Ice Bucket Challenge task shown in Figure 2.11.

**Figure 2.11 • The Ice Bucket Challenge task**

Ice Bucket Challenge

Throughout the summer of 2014, having a bucket filled with ice and cold water dumped on someone's head became pretty popular. Everyone across the country was getting involved with this activity! But why were they doing this?

In order to raise awareness for the ALS (Amyotrophic Lateral Sclerosis) Association, people would challenge others to either donate money to the organization within 24 hours, or they would have to dump water on their heads. The creators of the Ice Bucket Challenge believed that if three new people were challenged every time, eventually, billions across the country would donate and know about the ALS Association.

At Stage One of the process, a person challenged three others to take part in the Ice Bucket Challenge. At Stage Two, each of these three people challenged three others. How many people participated at Stage Five? How many people participated at Stage 10? Describe a function that would model the Ice Bucket Challenge process at any stage. Explain how your function models the situation.



PAUSE AND CONSIDER

What characteristics of the Ice Bucket Challenge task make it a good choice for the introductory lesson in a unit on exponential functions?

Because it cannot be solved by using a known rule or procedure, thus requiring students to reason and problem solve, the Ice Bucket Challenge is a doing-mathematics task. The task has other characteristics that make it a good choice for the lesson as previously described:

- It can be entered and solved in several different ways (e.g., draw a picture, make a table, use words to describe what is happening).
- It requires students to use and make connections between different representations (i.e., the context and a written or symbol rule).

- It asks students to explain their thinking.
- It asks students to generalize by describing a function that would model the process at any stage.

While creating tasks is certainly an option, it is a challenging endeavor! If you decide to create a task yourself, we encourage you to ask others to review and solve the task so that you can identify any possible pitfalls before you give it to students. This is exactly what Ms. Haines did with the Ice Bucket Challenge task!

Ensuring Alignment Between Task and Goals

Another challenge teachers often face is making sure that there is alignment between the task and the goal. That is, ensuring that the task they have selected as the basis for instruction provides students with the opportunity to explore the mathematical ideas that are targeted during the lesson.

Suppose, for example, you are teaching a lesson about slope. You decide that you want students to understand that slope is the ratio between a change in one variable relative to a corresponding change in another variable. You select a task that asks students to find the slope of lines given two points (e.g., (1, 6) and (3, 9)) without graphing. This basic *naked numbers* task does not provide students the opportunity to develop any understanding of what slope means. By eliminating graphing as a possible solution path, students are left to apply the formula for calculating slope $\left[\frac{(y_2 - y_1)}{(x_2 - x_1)} \right]$. As a result, students may end up with correct solutions but with limited understanding regarding the meaning of slope. In this situation, you have established a *learning* goal but have paired it with a low-level task that requires only application of a known procedure.

A mismatch between tasks and goals can also occur in a less extreme way. For example, Mr. Tanner initially selected the downloading music task, shown in Figure 2.7, for his lesson on systems of equations we discussed in the last section. Mr. Tanner selected this task because he thought students would relate to the context, and he liked the idea that students would have to explain their reasoning and would have to find the point of intersection to answer the question. Since this was an introductory task, and students had not yet learned how to solve systems algebraically, students would be able to use tables (with or without writing an equation first) and graph by hand.

So what's the mismatch? In the original version of the task, TUNE IN charged 99¢ per song (plus the additional \$8.00 per month membership charge). The point of intersection of the system in the downloading



music task (8.163, 16.082) would be nearly impossible to determine with the strategies to which students currently had access. While Mr. Tanner may ultimately want students to deal with *messy* numbers and be able to interpret what such a point means in context, such messiness could be reserved for a time when students have access to algebraic strategies and/or graphing calculators. But for now, if students could not find the point of intersection with some degree of accuracy, they would have difficulty achieving the lesson goals.

In this situation, it was an easy fix. After some discussion, Mr. Tanner changed the per-song charge for TUNE IN from 99¢ to \$1.00 as shown in Figure 2.7. This minor change made the task more accessible to students given their current knowledge and experience and made it possible to achieve the learning goals that Mr. Tanner had established.

Ensuring alignment between your goals and task is essential and the foundation on which to begin to engage in the five practices. If you find that your task does not fit your goals, consider the ways in which you can modify the task to provide more opportunities for students to think and reason as we described previously.

Launching a Task to Ensure Student Access

Launching or setting up a task refers to what the teacher does prior to having students begin work on a task. While it is not uncommon to see teachers hand out a task, ask a student volunteer to read the task aloud, and then tell students what they are expected to produce as a result of the work on the task, research suggests that attention to the way in which the task is launched can lead to a more successful discussion at the end of the lesson. Jackson and her colleagues (Jackson, Shahan, Gibbons, & Cobb, 2012) describe the benefits of an effective launch:

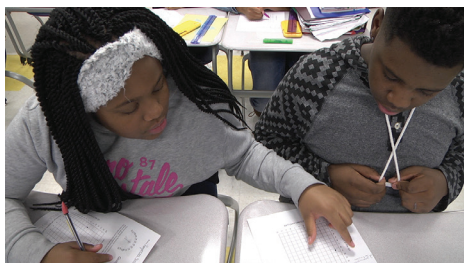
Students are much more likely to be able to get started solving a complex task, thereby enabling the teacher to attend to students' thinking and plan for a concluding whole-class discussion. This, in turn, increases the chances that all students will be supported to learn significant mathematics as they solve and discuss the task (p. 28).

So what constitutes an effective launch? In Analyzing the Work of Teaching 2.1, you will explore Mrs. Mossotti's launch of the State Fair task. We invite you to engage in the analysis of a video clip and consider the questions posed before you read our analysis. [NOTE: While the launch of a task occurs during instruction, it is planned for prior to instruction. Rather than describing what Mrs. Mossotti intended to do, we decided to take you into her classroom so that you could see for yourself!]



Analyzing the Work of Teaching 2.1

Launching a Task



Video Clip 2.1

In this activity, you will watch Video Clip 2.1 from Jennifer Mossotti's eighth-grade class.

As you watch the clip consider:

- What did the teacher do to help her students *get ready* to work on the State Fair task?
- What did the teacher learn about her students that indicated they were ready to engage in the task?
- Do you think the time spent in launching the task was time well spent?



Videos may also be accessed at
resources.corwin.com/5practices-middleschool

Launching a Task — Analysis

The first part of Video Clip 2.1 focused on ensuring that students understood the context of the problem. In fact, Mrs. Mossotti learned that all but one of her students (one who was new to the class that very day) had actually attended the fair. Since the Kentucky State Fair featured in the task paralleled the New York State Fair in some ways (e.g. ride tickets are purchased after entering the fair), students' real-world knowledge would not be in conflict with the task they would be working on.

Mrs. Mossotti made the decision to show the video clip of her son Mason on his first ride at the fair in an effort to capture students' attention and motivate them to work on the task. The students in her class know her three children, and she often tells stories about them in class. Prior to the lesson, Mrs. Mossotti described her plan:

So, I am launching it with a video of my middle child, who is by far my most difficult child. Sometimes I will refer to some of the students in my class as the Mason of the class that day. The video is



TEACHING TAKEAWAY

Taking the time to build relationships with students helps them feel more connected and can support their learning.

going to be Mason going on his very first ride at the New York State Fair. He is petrified. Then literally a second into the ride, he was very, very happy.

While it may strike you as unusual that Mrs. Mossotti shared a video clip of her son with the class, sharing aspects of her personal life in class helped her in building relationships with her students. Students are often more motivated to participate and learn when they feel a personal connection to the classroom (Horn, 2017).

In the next part of the launch, Mrs. Mossotti hands students a copy of the first page of the State Fair task, which contains the graph (without the questions that follow). Here she is focused on ensuring that the students could read the graph. By asking students to “write down two things that you notice,” Mrs. Mossotti gave all students access to the task (everyone would be able to say something), and their responses would help her determine whether students could identify key features of the graph.

Students’ responses revealed that they recognized that the graph showed the number of tickets and the amount of money spent, that the amount spent increased as the number of tickets increased, that the maximum number of tickets shown on the x -axis was 11 and the maximum cost shown on the y -axis was \$14.00, and that 10 tickets for \$13.00 was one point on the graph. The teacher’s role during this time was to record what students said. Although she intervened to ask a clarifying question as needed, she did not comment on the students’ contributions.

It is also worth noting that Mrs. Mossotti recorded students’ initials next to their contributions. In so doing, she was giving students *credit* and ownership of the idea. By publically acknowledging her students in this way, she was helping them build their mathematical identities and giving them an opportunity to be seen by their peers as competent.

As a result of the launch, Mrs. Mossotti learned that students could relate to the context of the problem and that they were able to read and make sense of the graph, which was central to the task. She did not give out the questions that went with the task initially because she did not want students to get too far ahead or give too much away. Prior to the lesson, Mrs. Mossotti indicated, “I’m not going to give them any mathematical knowledge or background in terms of the math for the task itself.” She wanted to make sure that students would have to think and reason their way through the problem.

Was the time spent launching the task time well spent? We could argue that it was essential to ensuring that students understood enough about what they were being asked to do to begin their work on the task. Too often, students are given a task and do not understand some aspect of it. When this occurs, the teacher then ends up moving from one group to the next answering questions that could have been cleared up with a more comprehensive launch.

TEACHING TAKEAWAY

A comprehensive launch ensures all students understand the entirety of the task, so that time is not spent later answering clarifying questions from groups or individuals.

Jackson and her colleagues (2012, p. 26) list “four crucial aspects to keep in mind when setting up complex tasks to support all students’ learning”:

1. Key Contextual Features of the Task
2. Key Mathematical Ideas of the Task
3. Development of Common Language
4. Maintaining the Cognitive Demand

Mrs. Mossotti launch embodied most of the features described by Jackson and colleagues. She made sure that students understood the context of the State Fair task (1), that they were able to read and make sense of the graph (2), and that the cognitive demand of the task was maintained by not suggesting a pathway to follow or giving away too much information (4). She did not explicitly distinguish between the entry fee and the amount paid for each ticket because this is at the heart of what she wanted students to figure out. Developing a common language (3) is often needed when there is vocabulary used in the task that might not be familiar to some or all of the students. For example, in the Downloading Music task (see Figure 2.7), you would want to make sure that all students understood what *downloading* meant and could describe it in their own words. In the State Fair task, however, this did not appear to be necessary.

Perhaps the most challenging part of launching a task is making sure that you do enough to ensure that students understand the context and what they are being asked to do but not so much that there is nothing left for students to figure out. For example, in the Downloading Music task, you would not want to provide students with a table and tell them to calculate the cost of buying one to 10 songs for each service. This would lower the demand of the task by providing a strategy for students to use and limit their opportunity to figure out what to do and how. In the Pizza Party task (Figure 2.6), you would not want to tell students that it was a division problem because that is something you want them to determine.

Conclusion

In this chapter, we have discussed the importance of setting clear goals for student learning and selecting a task that is aligned with the goal, and we have described what is involved in these practices and the challenges associated with them. Our experience tells us that if you do not take the time to seriously consider Practice 0 as a first step in carefully planning your lesson, the remainder of the practices will be built on a shaky foundation.

Mrs. Mossotti’s work in setting a goal and selecting a task provided a concrete example of a teacher who thoughtfully and thoroughly engaged

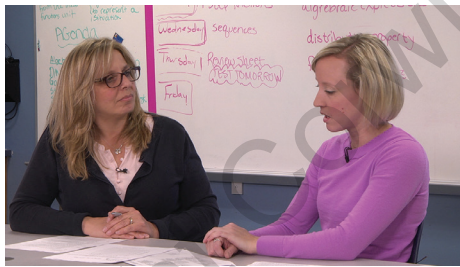


in this practice. Engaging in this practice in a deep and meaningful way does not happen overnight. It takes time and practice. As Mrs. Mossotti said, “Even if it feels like a failure the first time, or even if it feels like it’s taking a lot more time than you anticipated, that time is going to be earned back when students have that conceptual understanding. . . .”

Mrs. Saroney and Mr. Tanner’s efforts to determine what students would learn during their lesson and to ensure that the goals and tasks align made salient the challenges that teachers can face and overcome when engaging in this practice. In both cases, working with colleagues helped these teachers make progress in Practice 0.



Setting Goals and Selecting Tasks—Summary



Video Clip 2.2

To hear and see more about setting goals and selecting tasks, watch Video Clip 2.2.



Videos may also be accessed at
resources.corwin.com/5practices-middleschool

In the next chapter, we explore the first of the five practices: *anticipating*. Here, we will return to Mrs. Mossotti’s lesson and consider what it takes to engage in this practice and the challenges it presents.

Linking the Five Practices to Your Own Instruction

SETTING GOALS AND SELECTING TASKS

Identify a mathematical idea that you will be teaching sometime in the next few weeks. Working alone or with your colleagues:

1. Determine what it is you want students to learn about mathematics as a result of engaging in the lesson. Be as specific as possible. It is okay to indicate what students will do during the lesson, but do not stop there!
2. Select a high-level cognitively demanding *doing-mathematics* task that is aligned with your goals. Make sure that there are different ways to enter and engage with the task. Identify resources that are likely to help students as they work on the task.
3. Identify what students will say and do that will indicate that they are meeting the goals you have established.
4. Plan a launch that takes into account the four crucial aspects identified by Jackson and her colleagues.

Reflect on your planning so far. How does it differ from how you have previously thought about goals and tasks? In what ways do you think the differences will matter instructionally?