

WHAT YOUR COLLEAGUES ARE SAYING . . .

“This book provides the perfect answer to the question, ‘How can I help students engage in high-quality math discourse in my classroom?’ The experiences of real teachers in real classrooms, brought to life through a series of vignettes, provide vivid illustrations of how the 11 techniques described can get students thinking and talking about mathematics. The book is a game changer for elementary teachers!”

Margaret (Peg) Smith,

Emeritus Faculty, University of Pittsburgh

“We’ve come a long way since discussion in math class meant that individual students shared their strategies one after the other with little interaction or reflection. This book is based on the premise that discourse skills can and must be learned and practiced if all students are to have access to participation in high-quality talk about significant mathematical ideas. Based on a decade of work with teachers and coaches, it provides clear, specific strategies illustrated with classroom examples for supporting students as they learn how to talk, listen, and question during all phases of the math lesson.”

Susan Jo Russell,

Senior Researcher, TERC

“Packed with powerful teaching ideas—there are so many excellent teaching strategies in this single book! A teacher could learn to implement one or two of these techniques and the book will have been worth its cost. It provided ideas that I wanted to try to implement RIGHT AWAY!”

Amanda Jansen,

Professor of Mathematics Education
University of Delaware

“This book does a great job of providing how-to steps that I was able to incorporate into my own practice. These techniques for discourse are appropriate for a wide variety of grade and skill levels. I especially appreciated the strategies for differentiation and for meeting the needs of emergent multilinguals.”

Tyler Erickson,

Fifth-Grade Teacher

“We teachers know students can talk. But teaching how to talk to further mathematical understanding is challenging. *Activating Math Talk* gave me strategies to guide students, even reticent ones, into meaningful mathematical discourse. It challenged me to be more purposeful in ‘opening spaces for students to surprise you.’ It changed the way I taught and listened to students, making me a better teacher, and helped me create an exciting, respectful classroom environment where my students gained confidence and competence in building shared mathematical understanding.”

Kim Zeugner,
Elementary Teacher

“Fostering a discourse-rich classroom is essential for emergent multilingual learners to develop deep understandings of mathematics. The authors provide the what, why, and how of developing meaningful learning communities through practical, research-based suggestions that teachers can take directly into their classrooms. The inclusion of excerpts from real classrooms allows us insights into the teachers’ and learners’ experiences as we learn how to center and foster language in the mathematics classroom. This book is a great resource for teachers and teacher educators who wonder how to help move the math forward while students are acquiring language.”

Zandra de Araujo,
Associate Professor of Mathematics Education,
University of Missouri

“This book is set up well for grade-level teams to do a book study and set goals for how they are working towards creating a discourse community in their classrooms.”

Joshua Males,
K–12 Mathematics Curriculum Specialist

“This is a needed resource right now. We teachers just aren’t doing this in our classrooms and we need a resource to develop this aspect of our instruction.”

Kyle Cayce,
Elementary Teacher

Activating Math Talk: The Book at a Glance

This book is about purposeful and explicit math talk techniques for the elementary classroom that promote high-quality discourse school-wide. The techniques align to each phase of a lesson (launch, explore, discuss), and particular attention is paid to engaging emergent multilingual learners in math talk. In these pages you'll find the following useful features.

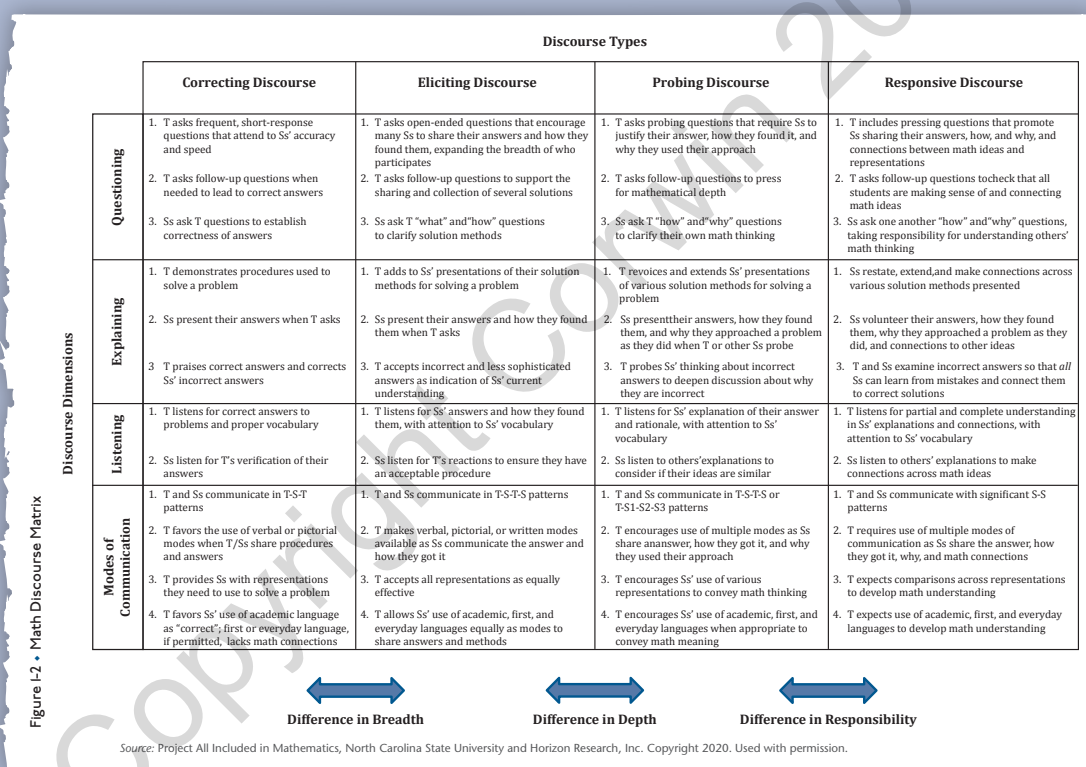


Figure 1-2 • Math Discourse Matrix

The Math Discourse Matrix describes the qualities of four discourse types and what teachers and students do in each.

Throughout the book, find definitions of key terms and opportunities to consider how key points or nuances relate to your own instruction.

math is defined as including “purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication” (p. 29). Students have opportunities to express ideas, clarify meaning, construct arguments, and compare approaches to build shared understanding of math concepts as well as flexibility with procedures.

Math discourse: patterned ways of using questioning, explaining, listening, and different modes of communication in the classroom to promote conceptual understanding in math for all learners

In our work, we think of math discourse as *patterned ways of using questioning, explaining, listening, and different modes of communication in the classroom to promote conceptual understanding in math for all learners.*

Because each part of the math discourse definition is important, we like to break it down. Let’s start with *patterned*. Classroom discourse is not about the way teachers do something one day or for a few cool lessons. Discourse is created out of the overall structures teachers put in place every day, sometimes without saying anything. Students learn how to participate from what teachers emphasize is valuable.

THINK ABOUT IT

Think about the proposed definition for math discourse.

What are existing conversation patterns in your math classroom? What do these patterns mean to you when it comes to helping your students learn mathematics?

Consider the following two tasks.

<p>Task 3-1</p> <p>Mr. Gayles is building a rectangular school garden with two 5-foot sides and two 7-foot sides. What is the perimeter of the garden?</p>	<p>Task 3-2</p> <p>The perimeter of the rectangular school garden Ms. Guilford is building is 24 feet. What are the possible dimensions of the garden?</p>
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Figure 3-1 • Does Your Task Promote or Limit Discourse?

DISCOURSE-PROMOTING TASKS	DISCOURSE-LIMITING TASKS
<ul style="list-style-type: none"> Require cognitive effort due to the newness or the unpredictable nature of the solution Require examination of task context that may limit or expand possible solution strategies and solutions Require access to relevant knowledge and experiences to work through the task Require examination of the nature of mathematical concepts, processes, and relations Require complex and nonalgorithmic thinking Require multiple representations and connections among representations 	<ul style="list-style-type: none"> Require a focus on the answer with attention to its correctness instead of its processes Require no explanation or explanation focused solely on definitions and procedure without relevance to context Require reproducing previously learned facts, rules, or formulae Require no connection to concepts or meaning that underlie definitions or procedures Require little cognitive demand beyond use of the procedure called for Require one specifically called-for representation

Source: Adapted from Smith and Stein (1998).

Learn how to analyze how well your math tasks promote or limit math discourse, and find many examples of high-quality tasks.

Examine Practice scenarios provide opportunities to analyze and reflect on the implementation of talk techniques in authentic elementary classrooms.

EXAMINE PRACTICE



Read the following vignette from Ms. Ladeaux's classroom, where Story Problem Retelling is used two times during the Launch phase of a lesson. What purposes might Ms. Ladeaux have for each use of the technique?

Story Problem Retelling in Ms. Ladeaux's Kindergarten Classroom

Planning the Lesson

After 13 years teaching first grade, I am in my first year teaching Kindergarten. My story shares my experience launching a lesson on addition and subtraction within 10 using Story Problem Retelling. My class had been exploring the idea that adding or subtracting 1 gives the next, or the prior, number on the counting sequence. We also worked with adding or subtracting 2. So in this lesson I wanted to build on that learning as we worked with bigger numbers within 10. My math goal was for students to realize there are many strategies they can use to solve addition or subtraction problems within 10.

SIGNS OF SUCCESS



- Students use prior knowledge as an asset to make their predictions.
- Students listen carefully to each other's predictions.
- Students' bets contribute to the mathematical understanding of the problem.
- Students analyze predictions using the two key questions.

CAUTION SIGNALS



- Students make predictions that are unrelated to the problem or only about the context. (Use the "What new mathematical information" question to point out this limitation.)
- Students focus only on guessing what the problem will say, not on what makes an appropriate prediction. (Remind students what makes a prediction mathematically productive.)

Signs of Success and Caution Signals highlight indicators of successful technique implementation and foreshadow potential challenges that might arise.

Key Takeaways help summarize the most salient points from each lesson phase.

KEY TAKEAWAYS ABOUT THE LAUNCH PHASE



Part III presented three talk techniques for the Launch—Story Problem Retelling, Task Think-Aloud, and Math Bet Lines—and their accompanying classroom vignettes. These chapters illustrated teachers attending to different purposes in the Launch to prepare students for productive engagement with math content and in mathematical discussions. Here are the main takeaways from these chapters:

- Use the Launch techniques for specific purposes, and remember that techniques for techniques' sake do not change discourse.
- Set the tone for the work of the lesson with the Launch, and make sure your students can be successful in the subsequent Explore and Discuss phases.
- Do not use the Launch to tell students what to do, and avoid going so far with your Launch that there is little left for students to do or talk about.



DISCUSS WITH COLLEAGUES

- 1 How does your definition of math discourse compare to the definition provided in this chapter? Which of the four parts of the definition (patterned; using questioning, explaining, listening, and different modes of communication; conceptual understanding; for all learners) are easier for you to support in your classroom? Which are more challenging? Why?
- 2 Think about a math lesson you recently taught. Share what happened in this lesson with your colleagues using the discourse features from the Math Discourse Matrix (Figure I-2). What evidence from your classroom indicates the types of discourse you and your students engaged with during the lesson?

Discuss With Colleagues sections help you reflect on your practice as a team.

Connect to Your Practice sections give you opportunities to apply and reflect on a new skill.



CONNECT TO YOUR PRACTICE

Pick one discourse dimension (questioning, explaining, listening, or modes of communication) under probing or responsive discourse. Plan and implement a math lesson focused on helping students engage in features of that particular dimension. Think about supports your students will need to engage in those ways. After your lesson, consider:

- ❑ How well did students engage in those features of the dimension? What was successful and what was challenging for students?
- ❑ What might you do differently in the future to improve student engagement in that dimension?

ACTIVATING Math Talk

Grades
K-5

11 Purposeful Techniques for Your Elementary Students

Paola Sztajn | Daniel Heck | Kristen Malzahn

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Preface

SETTING THE STAGE: SUPPORTING MATH LEARNING THROUGH PRODUCTIVE TALK

Talk has been recognized as key to learning for quite some time, and research continues to demonstrate the importance of productive talk for learning math. Presenting and justifying one's reasoning, critiquing the arguments of others, and engaging in meaningful mathematical discussions are fundamental practices that support math learning. Further, listening, communicating, and collaborating are skills we want to instill in our students as they grow up to fully participate in a technological society. So engaging students in productive talk is a win-win investment in math classrooms.

Almost a decade ago, we started our work with elementary teachers to support them as they engaged their students in math talk. Three important principles guided our initial efforts and have continued to inform us over the years.

Principles That Guide Our Work

1. Practical techniques that can be readily and purposefully implemented help teachers build a toolkit for activating high-quality math discourse.
2. Techniques help students learn about important and specific aspects of math talk, building skills and dispositions for participating in high-quality math discourse.
3. All students can learn to participate productively in high-quality math discourse when they are provided opportunities with scaffolds to support their engagement; the techniques we share with teachers create these opportunities.

PURPOSEFUL TALK TECHNIQUES FOR THE ELEMENTARY CLASSROOM

Searching for practical techniques to support discourse, we partnered with colleagues in literacy education and identified techniques that could be adapted for math instruction. We developed a yearlong professional development program around these adapted techniques and worked with over 300 K–2 teachers. Their success with the program and implementation of the talk techniques in their classrooms encouraged

us to share our work with elementary teachers more broadly. Teachers in our program learned to choose what technique to use when and for what purpose—something we emphasize, because techniques used for their own sake do not strengthen math lessons. Knowing when and why to use particular techniques is key, and in this book, we make the case for such purposeful use of talk techniques to activate and improve math discourse.

PROMOTING HIGH-QUALITY DISCOURSE SCHOOLWIDE

In addition to helping teachers think about their own teaching, we designed this book as a resource for promoting high-quality discourse schoolwide. The content of the chapters and discussion questions within them are meant to spark conversations among teachers, teacher leaders, and administrators about how to get kids talking about math in productive ways. Sharing these ideas with colleagues during professional learning opportunities can generate strong momentum at your school to improve the overall quality of discourse in math classrooms. Here are some suggestions for using this book to promote and nurture productive math talk schoolwide:

- **Professional learning community (PLC) book study:** Read and discuss this book during PLC meetings. Use the discussion questions at the end of each chapter as opportunities for application to practice and reflection.
- **Grade-level team planning:** Incorporate ideas and resources from the book into grade-level team planning to more purposefully plan for discourse.
- **Lesson study:** Use the lesson study process where teachers collaboratively plan, teach, observe, and debrief lessons aimed at high-quality discourse using talk techniques from the book.
- **Demonstration lessons:** Conduct demonstration math lessons that incorporate talk techniques and use the resources from the book to analyze the nature of student discourse that occurred and what supported or hindered it.
- **Classroom video analysis:** Analyze videos of teachers implementing talk techniques in the classroom, and discuss both the nature of the discourse that occurred and teacher moves that supported or hindered it (e.g., video clubs).

ORGANIZATION OF THIS BOOK

Parts I and II of this book provide important context and purpose to the techniques. Although you may be tempted to jump ahead to the technique chapters, we strongly encourage you to read Parts I and II first because they provide guidance to ground you in this work. The initial chapters help you consider what high-quality discourse looks like in math classrooms, how everyone participates, and what it takes to activate such talk. You'll also find suggestions for how to create a discourse community and how to plan your lessons to support robust math conversations. We pay deliberate attention to engaging all students, particularly emergent multilingual learners.

Parts III, IV, and V present the 11 talk techniques, organized by three phases of enacting a math lesson: Launch, Explore, and Discuss. The technique chapters also include vignettes from real classrooms illustrating the techniques in action. The book concludes with Part VI, which ties everything together and offers a lesson-planning tool to help you focus on purposeful planning and reflection when using the talk techniques in your instruction.

SUPPORTING FEATURES

To support professional learning and reflection on math discourse practices, we provide the following features in the chapters that invite you and your colleagues to explore the ideas presented and consider their application to your classroom instruction.

- **Definitions of key terms** in the margins
- **Think About It** moments to draw your attention to particular key points or nuances of practice and help you consider how they relate to your own instruction
- **Scenarios** that provide classroom examples of tasks and techniques
- **Signs of Success and Caution Signals** to highlight indicators of successful technique implementation and foreshadow potential challenges that might arise
- **Examine Practice** vignettes that provide opportunities to analyze and reflect on the implementation of talk techniques in authentic elementary classrooms; note, pseudonyms have been used for all teachers and students

- **Discuss With Colleagues** questions meant for group discussions to take stock of what you are reading and how it may impact math teaching and learning in your classrooms
- **Connect to Your Practice** activities for you to try in your own classroom and then reflect

Written in a friendly style with direct language that demystifies math talk and provides specific guidance for its implementation, this book is useful to a wide audience, from elementary school teachers, to leaders, to those interested in discourse or math instruction more broadly. It is the culmination of many years of work with teachers and leaders who have tested and vetted the ideas we share. As you get started, we encourage you not only to read the chapters in this book, but also to try the ideas in your practice and talk about them with your colleagues, because talk will help you learn!

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PART I

Understanding High-Quality Math Discourse for All Students

What does it mean to activate math talk? Our work with teachers has shown us that the answer to this question varies tremendously. Part I introduces our vision of what high-quality math discourse means to us, and that vision guides everything in this book. In Part I we discuss the following:

- What constitutes high-quality math discourse (Chapter 1)
- What it means to activate high-quality discourse for *all* students, in particular emergent multilingual learners (Chapter 2)
- What teachers need to know mathematically to be able to activate high-quality discourse for *all* students (Chapter 3)

Starting with some definitions to establish important vocabulary, Part I sets the stage for the talk techniques presented later in this book. Keep in mind that techniques are meaningless unless used in service of wider goals. Part I sets the vision for these goals; as you start to read about and implement the techniques in your classroom, continue to come back to these introductory chapters for reflection. You will see that teachers who allow students to talk about math open a new door for math instruction! Kids have incredible ideas that can transform teaching. Focusing on what students say generates new excitement in teaching math, and having techniques to do so can make productive math talk a reality in the classroom.

High-Quality Discourse in Math Classrooms

Kids talk. In fact, most kids talk most of the time when among classmates or friends. Students of all ages, including young ones, are often not shy during recess or while playing on the playground. They talk in the classroom, and teachers often have to ask them not to talk during instructional time. So what's all the fuss about getting kids to talk in class and engaging them in discourse when learning math? Wouldn't it just be the case that if we let students talk, then math talk would flourish in the classroom?

Activating math talk is not that simple and actually not that natural. This book is based on the premise that students need to *learn to talk in specific ways* during math lessons for the talk to contribute to math learning—and this is especially true for young students. These ways of talking are different from how kids talk on the playground or in other subject areas. Thus, activating math talk is not just a matter of telling students to talk. Rather, participation in productive math conversations is a skill that is taught and learned.

COMPARING DISCOURSE SCENARIOS

To start thinking about math discourse, consider the following two fictional scenarios constructed to highlight some important points. In each scenario, the teacher is working with a second-grade class on the following problem:

For all games at Poe Elementary School, students have to wear either blue or red shirts. There are 35 students in the gym for a game. If 16 are wearing red shirts, how many are wearing blue?

Students had time to think about the problem before the teacher initiated the discussion. As you read the scenarios, think about what makes the discourse in these classrooms similar or different. Consider what the teachers and the students are doing or saying in each classroom.

SCENARIO 1-1

Teacher: Who knows the answer?
(About five students raise their hands immediately and the teacher points to one of them.)

Student 1: Nineteen.

Teacher: Great, how do you know?

Student 1: Because 35 minus 16 is 19.

Teacher: Great, and how do you know that?

Student 1: I wrote it down here on my paper. (Student shows the paper.) I cannot take 6 away from 5, so I cross out the 3 to make 15. I went 7, 8, 9, 10, 11, 12, 13, 14, 15. (Student tracks numbers counted with fingers.) That is 9, so I need 9 to go from 6 to 15. The 3 is now a 2. Now I have 2 take away 1 and that is 1. The answer is 19.

Teacher: Very good. Did everyone get that? (Teacher waits a couple of seconds and a handful of students nod their heads yes.) Anyone else? Any other way of doing this?

Student 2: I do not get it because I think it is 20. I just counted: 16, 17, 18, 19, . . . (student continues to count and show fingers to keep track of each number counted) 34, 35. See, I went over my hands twice and counted 20.

Teacher: You do not count the 16. You start at the 17 because you are counting the jumps. Very nice strategy! If you start at 17, you will count 19 numbers, so the answer is 19. Good strategy. Other ideas? (Teacher waits a little more.) Can someone explain to me the two solutions we have discussed?

Student 3: The first one is just like 35 minus 16, and we know that is 19. The second one is like, you start counting up from 16 to get to 35, and that is also 19.

Teacher: Exactly, two different ways to solve this problem. Very nice. Anyone else?

Student 4: I used blocks and also got 19.

Teacher: How did you use your blocks? Can you show our class?

Student 4: I got 3 blocks here and then 5 blocks to make 35. (Student shows tens and ones blocks and continues using the blocks throughout this explanation.) I have to separate 16. I take 1 ten and 5 ones because I have them. I still have 20, but I have to take 1 more away because it is 6 and not 5. If I take that one more now I have 19.

Teacher: Very good. You went from the left to the right, and took 1 from 3 first, but it worked with the blocks. Good work. Let's look at the next problem.

SCENARIO 1-2

Teacher: Who knows how to solve this problem? (About five students raise their hands immediately and the teacher waits until more hands go up.) Let me see . . . someone who has not yet shared today. (Teacher waits a little longer, a couple more hands go up, and teacher points to one of them.)

Student 1: I got 19.

Teacher: Can you explain the problem you were trying to solve and then how you got 19?

Student 1: I was trying to do 35 minus 16.

Teacher: Why were you trying to do that?

Student 1: Because 35 is like red and blues together. The students are blue or red and 16 are red. I am trying to find the blue ones.

Teacher: (Looking at the class.) What do you think about this idea that to find the blue shirts we need to take apart the 16 from 35?

Student 2: Can a student have a white shirt?

Student 1: I was thinking that students with wrong shirts are not in the gym. I was thinking the 35 students had a red shirt or a blue shirt. No other colors.

Student 3: If there are white shirts it is really hard. And what about other colors? That is confusing.

Teacher: Good point, so, in this problem, we are thinking the shirts are

blue or red only. This is important. Let's hear the rest of the explanation. Why are you subtracting?

Student 1: I have to separate the 16 red from all students to see which ones are blue. So I wrote 35 minus 16 on my paper. (Student shows the paper.) I cannot take 6 away from 5, so I cross out the 3 and it is now 15.

Teacher: Let's stop here. Can you explain what you mean by you cannot take 6 away from 5?

Student 1: It is too many. I have to start with my ones and I am trying to subtract 6, but I only have 5 ones in the 35. So I have to get a ten from the 30 and make it into ones.

Student 4: Like if you had blocks.

Student 1: Yes, I ungrouped 1 ten and I have 2 tens and 15 ones. And I can now take 6 ones away. That is 9. And I can take 1 ten away, and I have 1 ten left, so 19 blue shirts left.

Teacher: Does anyone have a question about this solution?

Student 5: Could you trade all the tens?

Student 1: Hmm . . . I guess I could, but I only need to trade one of them and it is easier. I only need to subtract 6 ones.

Student 4: I added instead of subtracted, is that okay?

Teacher: What do you mean?

Student 4: I was adding to the 16. I added 10 and got to 26. Then I counted:

27, 28, . . . (student tracks numbers counted with fingers) 34, 35. That is 9. With the 10, I also got 19.

Student 6: I added too, but got 20. So 16, 17, . . . (student tracks numbers counted with fingers) 34, 35. That is 20 and not 19.

Student 4: You have to start with 17, because 16 is still red. Seventeen is the first blue shirt. Like if you lined them all up. Do you get it?

Student 6: Okay. If I start at 17, then I guess you are right, I get 19.

Teacher: Those of you who have not yet shared, what are you thinking?

Student 7: It looks like we have different ways and there are 19 students with blue shirts.

Teacher: What do you mean different ways?

Student 7: We can subtract, we can add tens and ones, or we can add all ones.

Teacher: Nice summary. Anyone else? When we add we are counting up, remember? (Points to one student who is quiet.) Can you summarize from the beginning?

Student 8: Hmm . . . there are 35 students in the gym. Sixteen have red shirts. We want to find out the blue shirts. There is like blue and red only. To find the blue, I can subtract 16 from 35 or count up from 16 to 35. If you count up, you start at 17. We get 19 blue shirts.

Student 9: I have another way to do it. I counted backward.

Teacher: That is great, and counting backward can also work—you can show me your way later. And can there be even some other ways to solve this? (Students nod yes.) I am sure there are a few other great ways. For today we will stop our discussion with that great summary because we need to move on to other topics, but we will come back to the counting backward strategy later this week.

Figure 1-1 lists a few features of the two scenarios that are important to consider when comparing the nature of the math talk happening. Check some of these against the ones you noticed regarding what the teachers and students were doing.

When thinking about different math classroom conversations, it is important to consider who is talking, who they are talking to, what is being talked about, and for what purposes. Are students explaining what they did, how, and why as part of their argument? Are they making connections across multiple ideas, procedures, or representations? Who is asking the questions, and what types of questions are they asking? Who is answering these questions?

THINK ABOUT IT

Compare and contrast Scenario I-1 and Scenario I-2.

What did you notice about the nature of math discourse in each scenario? How are they similar and different in terms of how students are participating in the discussion and what math they are learning? How are the teachers supporting the students?

Figure I-1 • Comparing Discourse Scenarios

	SCENARIO I-1	SCENARIO I-2
TEACHER	<ul style="list-style-type: none"> Includes more than one student in conversation. Focuses on solving $35 - 16$. Asks students to explain how they found an answer. Corrects students as needed. 	<ul style="list-style-type: none"> Includes more than one student in conversation, and purposefully includes students who might otherwise not participate. Attends to the meaning of the problem (35 total shirts, 16 are red, how many are blue). Focuses on developing appropriate use of place value language. Allows students to ask questions to each other. Encourages students to explain how and why for their solutions.
STUDENTS	<ul style="list-style-type: none"> Share their computation strategies. Share correct and incorrect strategies. Talk to the teacher. 	<ul style="list-style-type: none"> Share their computation strategies. Share correct and incorrect strategies. Explain the connection between strategies and the meaning of the problem. Ask each other questions. Talk to the teacher and to their classmates. Use appropriate place value language.

The two scenarios presented portray math classroom discourses that have several features in common, such as many students explaining their math work. The purposefulness of the two teachers in organizing the conversation, however, is very different. The teacher in Scenario 1-2 is more focused on

- supporting students in making meaning,
- promoting connections within procedures,
- engaging more students in the conversation, and
- refraining from talking after every student turn.

If the teacher's goal is to develop mathematical understanding for all students in the classroom, Scenario 1-2 represents a more inclusive and supportive type of discourse for instruction.

DEFINING MATH DISCOURSE

In *Principles to Actions* (National Council of Teachers of Mathematics, 2014), the facilitation of meaningful math discourse is listed as one of the key practices for effective teaching and learning. Discourse in

math is defined as including “purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication” (p. 29). Students have opportunities to express ideas, clarify meaning, construct arguments, and compare approaches to build shared understanding of math concepts as well as flexibility with procedures.

In our work, we think of math discourse as *patterned ways of using questioning, explaining, listening, and different modes of communication in the classroom to promote conceptual understanding in math for all learners*.

Because each part of the math discourse definition is important, we like to break it down. Let’s start with *patterned*. Classroom discourse is not about the way teachers do something one day or for a few cool lessons. Discourse is created out of the overall structures teachers put in place every day, sometimes without saying anything. Students learn how to participate from what teachers emphasize is valuable. This is why teachers’ purposefulness in setting norms for discourse matters. Over time, classrooms develop expected and shared patterns for discourse that become stable. This pattern of engagement during classroom interactions is what we are calling classroom discourse.

The second part of the definition is *using questioning, explaining, listening, and different modes of communication*. This clarifies that discourse is made of the questions asked, the explanations accepted, the ways in which teachers and students are listening to each other, and the types of language and nonverbal tools used to present and represent ideas. For example, in Scenario 1-1 the teacher accepts it when Student 1 says that the solution is 35 minus 16 and goes on to say, “I cannot take 6 away from 5, so I cross out the 3 to make 15.” In Scenario 1-2, the teacher requires the student to explain *why* subtraction would work to solve the problem and also interrupts some answers to encourage clear and appropriate use of place value terminology. These teachers are listening to and accepting different types of answers from students. Over time, students internalize these messages from their teachers, including what are good questions, good explanations, and useful representations.

The next important part in this definition is *conceptual understanding*. This means that students are learning more than how to carry out

Math discourse: patterned ways of using questioning, explaining, listening, and different modes of communication in the classroom to promote conceptual understanding in math for all learners

THINK ABOUT IT

Think about the proposed definition for math discourse.

What are existing conversation patterns in your math classroom? What do these patterns mean to you when it comes to helping your students learn mathematics?

procedures with accuracy; they understand the math behind those procedures. Of course we want all students to learn procedures and develop fluency with them. But procedural fluency builds from conceptual understanding and requires efficiency and flexibility. Like in literacy, if a student can sound out a word but cannot comprehend its meaning or use it appropriately, the student is not yet reading. A good example of lack of conceptual understanding comes from an old national assessment item (Carpenter, Lindquist, Matthews, & Silver, 1983) adapted here to use easier numbers. The question asked, “An army bus holds 35 soldiers. If 400 soldiers are being bused to their training site, how many buses are needed?” Many students answered that 11 or “11 remainder 15” buses were needed, which demonstrates procedural solutions. A student with conceptual understanding of division and remainders can find the solution of 11 remainder 15 and knows that 12 buses are needed to seat all 400 soldiers.

The final part of our definition is *for all learners*. All means each and every one. Teachers are responsible for the learning of every child in the classroom, and discourse has to take all of them into consideration. Over time, there cannot be students in the classroom who are consistently excluded from participating. Remember the patterns? Students can participate in different ways on different days, respecting their own identities and areas of expertise. But having students who are seldom engaged or who are rarely asked to answer high-level math questions is a problem! So attention to the patterns of accepted engagement expected from, and actually taught to and learned by, each and every student is key for understanding and activating math talk in the classroom.

DIFFERENT TYPES OF MATH DISCOURSE

Because there can be several different patterned ways of using questioning, explaining, listening, and modes of communication in the classroom, we contend that teachers can use different types of math discourse in the classroom. These types can be used at different times and for different purposes. Figure 1-2 describes four types of discourse that are commonly seen in math classrooms.

Each cell of the Math Discourse Matrix contains indicators of what teachers (T) and students (S) are doing during a particular type of classroom discourse.

Figure I-2 ♦ Math Discourse Matrix

Discourse Dimensions		Discourse Types			
		Correcting Discourse	Eliciting Discourse	Probing Discourse	Responsive Discourse
Questioning	<ol style="list-style-type: none"> 1. T demonstrates procedures used to solve a problem 2. Ss present their answers when T asks 3. T praises correct answers and corrects Ss' incorrect answers 	<ol style="list-style-type: none"> 1. T adds to Ss' presentations of their solution methods for solving a problem 2. Ss present their answers and how they found them when T asks 3. T accepts incorrect and less sophisticated answers as indication of Ss' current understanding 	<ol style="list-style-type: none"> 1. T asks probing questions that require Ss to justify their answer, how they found it, and why they used their approach 2. T asks follow-up questions to press for mathematical depth 3. Ss ask T "how" and "why" questions to clarify their own math thinking 	<ol style="list-style-type: none"> 1. T includes pressing questions that promote Ss sharing their answers, how, and why, and connections between math ideas and representations 2. T asks follow-up questions to check that all students are making sense of and connecting math ideas 3. Ss ask one another "how" and "why" questions, taking responsibility for understanding others' math thinking 	
Explaining	<ol style="list-style-type: none"> 1. T listens for correct answers to problems and proper vocabulary 2. Ss listen for T's verification of their answers 	<ol style="list-style-type: none"> 1. T listens for Ss' answers and how they found them, with attention to Ss' vocabulary 2. Ss listen for T's reactions to ensure they have an acceptable procedure 	<ol style="list-style-type: none"> 1. T listens for Ss' explanation of their answer and rationale, with attention to Ss' vocabulary 2. Ss listen to others' explanations to consider if their ideas are similar 	<ol style="list-style-type: none"> 1. T listens for partial and complete understanding in Ss' explanations and connections, with attention to Ss' vocabulary 2. Ss listen to others' explanations to make connections across math ideas 	
Listening	<ol style="list-style-type: none"> 1. T and Ss communicate in T-S-T patterns 2. T favors the use of verbal or pictorial modes when T/Ss share procedures and answers 3. T provides Ss with representations they need to use to solve a problem 4. T favors Ss' use of academic language as "correct"; first or everyday language, if permitted, lacks math connections 	<ol style="list-style-type: none"> 1. T and Ss communicate in T-S-T patterns 2. T makes verbal, pictorial, or written modes available as Ss communicate the answer and how they got it 3. T accepts all representations as equally effective 4. T allows Ss' use of academic, first, and everyday languages equally as modes to share answers and methods 	<ol style="list-style-type: none"> 1. T and Ss communicate in T-S-T or T-S1-S2-S3 patterns 2. T encourages use of multiple modes as Ss share answers, how they got it, and why they used their approach 3. T encourages Ss' use of various representations to convey math thinking 4. T encourages Ss' use of academic, first, and everyday languages when appropriate to convey math meaning 	<ol style="list-style-type: none"> 1. T and Ss communicate with significant S-S patterns 2. T requires use of multiple modes of communication as Ss share the answer how they got it, why, and math connections 3. T expects comparisons across representations to develop math understanding 4. T expects use of academic, first, and everyday languages to develop math understanding 	
Modes of Communication					

Difference in Breadth



Difference in Depth



Difference in Responsibility



Source: Project All Included in Mathematics, North Carolina State University and Horizon Research, Inc. Copyright 2020. Used with permission.

When engaging their students in these different types, teachers have different goals. For example:

THINK ABOUT IT

Before moving forward, spend some time examining the Math Discourse Matrix (Figure I-2).

What are students and teachers doing in each type of discourse in terms of their questioning, explaining, listening, and modes of communication?

- Correcting discourse can be appropriate for practicing facts.
- Eliciting discourse can support many students in joining the conversation.
- Probing and responsive discourse can develop conceptual understanding and build procedural fluency from this understanding.
- Responsive discourse can support students in taking responsibility for their learning.

We will take a more careful look at each of these discourse types.

Correcting Discourse

This type of classroom discourse is organized around the teacher initiate–student respond–teacher evaluate (IRE) pattern of discourse in which the teacher asks questions, a student responds (*what* they did or found), and the teacher listens to verify whether the answer is right or wrong. The teacher then moves to accept the answer as correct, or corrects the student and provides the answer, or asks a new question or a different student for the correct answer. For teachers who may have learned math through engagement with this type of discourse, it can become a default pattern to which they turn. This type of discourse can be effective to *access and assess students' accuracy and speed* regarding factual math knowledge and supports recall of facts and procedures. Correcting discourse lacks attention to students' own strategies and does not explicitly promote student engagement with strategic competency, math concepts, or higher-order thinking.

Correcting Discourse:

a type of discourse that follows the pattern of teacher asks, students respond (*what*), and teacher verifies the correctness of the answer. It can support speed and accuracy with facts and procedures.

Eliciting Discourse

The transition between correcting and eliciting classroom discourse involves a *difference in breadth* of what is discussed and by whom. This type of discourse can include a change in turn-taking patterns and wait time so that more students participate in the classroom discourse community, expanding the breadth of who is included in the conversation and what is discussed. The teacher collects several answers to a problem, and students present their mathematical solutions together with explanations of their procedures (*what* and *how*). In this type of discourse, the teacher asks open-ended questions

Eliciting Discourse:

a type of discourse in which the teacher elicits and welcomes participation from a broad group of students who share their solutions (*what* and *how*). It can support engagement in math discourse.

and creates a safe space for students' mathematical thinking. Students feel comfortable knowing that all answers are welcomed and mistakes become nonshameful events. Equally valuing all students' solutions can sometimes mean that less sophisticated mathematical answers, and sometimes even incorrect answers, remain unchallenged and more sophisticated and conceptually rich answers remain unexplored.

Probing Discourse

The transition between eliciting and probing classroom discourse involves a *difference in depth* of the mathematical conversation. Here the teacher transitions from eliciting a collection of student answers to probing students' mathematical thinking and showing appreciation for their mathematical justifications and strategic competence (*what, how, and why*). While staying positive and supporting a high level of student participation, the teacher uses questioning to probe for student explanations about their ideas or solutions, including why they were thinking or working in particular ways and what their ideas or solutions mean. The teacher requires students to construct and present their mathematical arguments, with justification. The teacher also encourages students to critique their peers' reasoning while positioning incorrect or partially correct ideas as learning opportunities on which to build. There is a change in what is accepted as mathematical justification and what it means to be engaged in doing math.

Probing Discourse: a type of discourse in which the teacher uses questions to probe students' answers and press for depth in students' explanations of their mathematical thinking (*what, how, and why*). It can support understanding and fluency with mathematical ideas.

Responsive Discourse

The transition between probing and responsive classroom discourse involves a *difference in responsibility* within the classroom organization. The teacher moves from being the sole authority for the quality of the content and the nature of the discourse to helping students take responsibility for them. The teacher purposefully works on releasing responsibility for the discourse to students. In turn, students understand that, together with the teacher, they are in charge of helping each other understand math. Maintaining both the eliciting and the probing nature of the two previous types of discourse, the teacher who engages with responsive discourse poses challenging tasks to students and asks them to not only present their thinking and justifications, but also establish mathematical connections among different solutions (*what, how, why, and connections*). The teacher expects *all* students to take initiative and to feel responsible for asking each other probing math questions that make thinking and justification available for discussion.

Responsive Discourse: a type of discourse in which students take responsibility for asking each other questions that probe their answers and press for explanations, establishing connections among different mathematical representations (*what, how, why, and connections*). It supports reasoning and strategic thinking.

Students become accustomed to comparing and contrasting their mathematical approaches to solving problems, examining similarities and differences across their solutions, and looking for connections. Through these collective, content-rich, and goal-focused math conversations, responsive discourse supports students' development of rigorous math knowledge, including conceptual understanding, procedural fluency, and strategic competence.

HIGH-QUALITY MATH DISCOURSE

From the definitions of the different types of discourse, we can see that high-quality discourse supports the development of all strands of math proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Research Council, 2001). High-quality discourse is purposeful and engages students in taking responsibility for their own learning and for the learning of their peers. Although high-quality discourse can include a combination of all types of discourse for appropriate purposes, to support the development of conceptual understanding, probing and responsive discourse need to become the most common and evident patterns in the classroom.

This book focuses on how to move in this direction. With appropriate classroom structures and techniques, teachers can teach all young learners how to engage in responsive discourse—we have seen it emerge and persist in the classrooms of teachers who have collaborated with us.

NOTES



DISCUSS WITH COLLEAGUES

- 1 How does your definition of math discourse compare to the definition provided in this chapter? Which of the four parts of the definition (patterned; using questioning, explaining, listening, and different modes of communication; conceptual understanding; for all learners) are easier for you to support in your classroom? Which are more challenging? Why?
- 2 Think about a math lesson you recently taught. Share what happened in this lesson with your colleagues using the discourse features from the Math Discourse Matrix (Figure I-2). What evidence from your classroom indicates the types of discourse you and your students engaged with during the lesson?



CONNECT TO YOUR PRACTICE

Pick one discourse dimension (questioning, explaining, listening, or modes of communication) under probing or responsive discourse. Plan and implement a math lesson focused on helping students engage in features of that particular dimension. Think about supports your students will need to engage in those ways. After your lesson, consider:

- How well did students engage in those features of the dimension? What was successful and what was challenging for students?
- What might you do differently in the future to improve student engagement in that dimension?