BIG IDEA 3

Reasoning About Addition and Subtraction of Fractions

TASK 3A

Tell if the DIFFERENCE of each will be more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

$\frac{9}{8} - \frac{1}{6}$ $\quad$ $\frac{3}{4} - \frac{6}{12}$ $\quad$ $\frac{9}{10} - \frac{1}{5}$ $\quad$ $\frac{6}{10} - \frac{8}{16}$

Choose one problem from above. Explain how you know the difference is more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

About the Task

How do we reason about sums and differences? Occasionally, we consider the amount that the minuend represents. In other situations, we think about the size that we take away. In this task, students are asked to consider subtraction results. The second part of the task provides insight into their reasoning. Some students can develop this reasoning through discussion and symbolic representations. Others benefit from discussion that makes use of concrete models or visual representations.

Anticipating Student Responses

Some of our students will find exact answers to determine if the difference is more or less than $\frac{1}{2}$. This is not the intention of the task. The reasoning in the third prompt is most easy to relate to common denominators. It makes sense that our students would make use of that strategy, but we would prefer that it be done through recall and relational understanding rather than computation. In the other prompts, we want students to reason about the minuend and subtrahend. The first prompt takes a small quantity away from a value greater than 1, or $\frac{9}{8}$. We know that this difference will be greater than $\frac{1}{2}$. In the second and fourth prompts, we take $\frac{1}{2}$ away from a value less than one whole. When this happens, our result is less than $\frac{1}{2}$.

PAUSE AND REFLECT

- **How does this task compare to tasks I've used?**
- **What might my students do in this task?**

Visit this book's companion website at **resources.corwin.com/minethegap/6-8** for complete, downloadable versions of all tasks.

WHAT THEY DID

Student 1

Student 1 correctly compares the difference in the first prompt. His other comparisons are incorrect. His writing implies that he might not know the mathematical meaning for the term "difference." It also reveals inaccurate perceptions about comparison ("$\frac{6}{12}$ is greater than $\frac{3}{4}$"). With these ideas in mind, it is unlikely that his correct response to the first prompt is representative of his understanding of fractions.

Student 2

Student 2's responses are correct. His work captures all of the procedures he used to find his comparisons. His work is accurate. Is it efficient? Is it based on reasoning about the meaning of the fractions being computed? Does he reason about the size of these fractions as well as their relationship to $\frac{1}{2}$? His writing provides an interesting note about subtracting $\frac{6}{10} - \frac{8}{16}$ by thinking of $\frac{8}{16}$ as $\frac{5}{10}$.

Student 3

Student 3's use of precise fractions and his writing signal procedurally based thinking. In fact, his writing talks about *how* he computed.

Student 4

Student 4 offers the reasoning the task is intended to provoke. He accurately compares the differences to $\frac{1}{2}$. There is no evidence of computation. He relates the minuend and the subtrahend to the $\frac{1}{2}$ benchmark. We should note that his reasoning is slightly flawed.

USING EVIDENCE

What would we want to ask these students? What might we do next?

Student 1

Student 1 presents different challenges. Does he have incorrect responses because of the inability to compare or the inability to subtract? Does he understand that we can work with fractions greater than 1? In these situations, we have to find ways to drill down to a root cause. We can make use of other, skill-based tasks to get a better sense of the student's understanding. In this case, we should verify that he understands the meaning of a fraction both greater and less than 1. We should then confirm that he can compare fractions with like denominators and then unlike denominators. From here, we can determine his ability to subtract fractions.

Students 2 and 3

We can be confident that Students 2 and 3 can rely on calculations to find solutions. Our next move is to work to develop their reasoning about the fractions being computed. We should revisit benchmark fractions and how they can support computation. As with other reasoning activities, it is critical that we provide students with opportunities to share and listen to strategies and approaches. During conversations, it is important to highlight clever approaches such as Student 2's swapping of $\frac{8}{16}$ with $\frac{5}{10}$.

Student 4

As noted, Student 4's approach for comparing the difference to $\frac{1}{2}$ is slightly flawed. He seems to consider the impact of removing half from a value less than 1 by noting that $\frac{6}{10}$ is greater than $\frac{1}{2}$ but should instead note that $\frac{6}{10}$ is less than 1. With this in mind, Student 4 still encourages us about the reasoning potential of our students. He, too, will benefit from conversations about strategies and logic when working with these sorts of tasks. We should praise his insight, but be sure to further the discussion by challenging his logic as noted here.

TASK 3A: Tell if the DIFFERENCE of each will be more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

$\frac{9}{8}-\frac{1}{6}$ $\quad$ $\frac{3}{4}-\frac{6}{12}$ $\quad$ $\frac{9}{10}-\frac{1}{5}$ $\quad$ $\frac{6}{10}-\frac{8}{16}$

Choose one problem. Explain how you know the difference is more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

Student Work 1

Tell if the DIFFERENCE will be more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

Subtraction problem	More than $\frac{1}{2}$ or less than $\frac{1}{2}$
$\frac{9}{8}-\frac{1}{6}$	More
$\frac{3}{4}-\frac{6}{12}$	More
$\frac{9}{10}-\frac{1}{5}$	less
$\frac{6}{10}-\frac{8}{16}$	More

Choose one problem from above. Explain how you know the difference is more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

$\frac{9}{8}-\frac{1}{6}$

$\frac{3}{4}-\frac{6}{12}$

$\frac{9}{10}-\frac{1}{5}$

$\frac{6}{10}-\frac{8}{16}$

The difference between $\frac{3}{4}$ and $\frac{6}{12}$ is that $\frac{6}{12}$ is greater than $\frac{3}{4}$ because you can't add it.

Student Work 2

Tell if the DIFFERENCE will be more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

Subtraction problem	More than $\frac{1}{2}$ or less than $\frac{1}{2}$
$\frac{9\times3}{8\times3}=\frac{27}{24}$ $\frac{1\times4}{6\times4}=\frac{4}{24}$ $\frac{27}{24}-\frac{4}{24}=\frac{23}{24}$ $\frac{9}{8}-\frac{1}{6}$	More
$\frac{3\times3}{4\times3}=\frac{9}{12}$ $\frac{9}{12}-\frac{6}{12}=\frac{3}{12}$ $\frac{3}{4}-\frac{6}{12}$	Less
$\frac{1\times2}{5\times2}=\frac{2}{10}$ $\frac{9}{10}-\frac{2}{10}=\frac{7}{10}$ $\frac{9}{10}-\frac{1}{5}$	More
$\frac{8}{16}=\frac{1\times5}{2\times5}=\frac{5}{10}$ $\frac{6}{10}-\frac{5}{10}=\frac{1}{10}$ $\frac{6}{10}-\frac{8}{16}$	Less

Choose one problem from above. Explain how you know the difference is more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

$\frac{6}{10}-\frac{8}{16}$ is an easy one. $\frac{8}{16}$ is equal to $\frac{1}{2}$ witch can convert to $\frac{5}{10}$ so I can easily subtract $\frac{6}{10}-\frac{5}{10}$ and my final answer is... $\frac{1}{10}$ witch is less than a half.

Student Work 3

Tell if the DIFFERENCE will be more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

Subtraction problem	More than $\frac{1}{2}$ or less than $\frac{1}{2}$
$\frac{9}{8}-\frac{1}{6}$	$\frac{23}{24}$ more
$\frac{3}{4}-\frac{6}{12}$	$\frac{3}{12}$ less
$\frac{9}{10}-\frac{1}{5}$	$\frac{7}{10}$ more
$\frac{6}{10}-\frac{8}{16}$	$\frac{6}{80}$ more

Choose one problem from above. Explain how you know the difference is more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

9/8 − 1/6 is more than ½ because if you find the common denomonator it will be 24. It will be 24 because $\frac{9\times3}{8\times3}$ and $\frac{1\times4}{6\times4}$ make $\frac{27}{24}-\frac{4}{24}$ which eaquals $\frac{23}{24}$ and $\frac{23}{24}$ is closer to 1 whole.

Student Work 4

Tell if the DIFFERENCE will be more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

Subtraction problem	More than $\frac{1}{2}$ or less than $\frac{1}{2}$
$\frac{9}{8}-\frac{1}{6}$	more
$\frac{3}{4}-\frac{6}{12}$	- Less
$\frac{9}{10}-\frac{1}{5}$	more
$\frac{6}{10}-\frac{8}{16}$	Less

Choose one problem from above. Explain how you know the difference is more than $\frac{1}{2}$ or less than $\frac{1}{2}$.

I Know $\frac{6}{10}-\frac{8}{16}$ is less because $\frac{6}{10}>\frac{1}{2}$ & $\frac{8}{16}=\frac{1}{2}$, so the difference is less.

OTHER TASKS

- **What will count as evidence of understanding?**
- **What misconceptions might you find?**
- **What will you do or how will you respond?**

Visit this book's companion website at **resources.corwin.com/minethegap/6-8** for complete, downloadable versions of all tasks.

TASK 3B: Which addition expression has the greater sum? Tell how you know which sum is greater.

$\frac{1}{4}+\frac{1}{2}$ or $\frac{1}{2}+\frac{1}{6}$ $\qquad$ $\frac{3}{4}+\frac{3}{2}$ or $\frac{3}{6}+\frac{3}{9}$

In this task, students compare sums of different fractions. Reasoning about both prompts likely relates the addends, the sums, or both to benchmarks. But there are other strategies. In the first prompt, both expressions add something to $\frac{1}{2}$ so the expression that adds more will yield a greater sum. In the second prompt, we might reason that the second sum is less than 1 while the first sum is clearly greater than 1 because one of the addends is greater than 1. In both prompts, students may show misunderstanding of how to add fractions, while others add correctly, though finding the results is much less efficient.

TASK 3C: Tell if the sum of each will be greater or less than 12. Choose one of the expressions and tell how you know it is greater or less than 12.

$3\frac{10}{12}+8\frac{5}{6}$ $\qquad$ $6\frac{1}{4}+5\frac{3}{8}$ $\qquad$ $6\frac{1}{2}+5\frac{1}{4}+\frac{1}{8}$ $\qquad$ $4\frac{3}{4}+4\frac{7}{8}+3\frac{1}{8}$

We can't limit our reasoning activities to fractions less than 1. In each prompt, students reason about the sum and compare it to 12. Students may decompose the mixed numbers to add wholes before considering the fractions. For example, in the first prompt we can add the whole numbers 3 and 8, finding a sum of 11. We're left with $11 + \frac{10}{12} + \frac{5}{6}$. Both fractions are much larger than $\frac{1}{2}$, so 11 plus those fractions will clearly be greater than 12. In the past, students who can procedurally add these mixed numbers may lead us to believe that they have a strong sense of fractions and computation when that may not have been the case.

TASK 3D: Henry says that he knows that the difference of $7\frac{7}{8}-2\frac{1}{2}$ is greater than 5 without subtracting on a piece of paper. How might he know this?

Henry also says that he knows the difference of $16\frac{2}{3}-5\frac{1}{6}$ is greater than 11 without subtracting on a piece of paper. How might he know this?

This task asks students to reason about the results of subtracting mixed numbers relative to whole numbers greater than 1. It also prompts them to avoid using a procedure or paper/pencil for finding the difference. This will prove to be a substantial challenge for some of our students. They may disregard the fractional part of each number, noting that $7\frac{7}{8}-2\frac{1}{2}$ isn't greater than 5 because 7 – 2 = 5. Others may mistake the fractional amount being taken away. Others will reason that $7\frac{7}{8}-2=5\frac{7}{8}$. They then share that $\frac{1}{2}$ from $5\frac{7}{8}$ is more than 5 because $\frac{7}{8}$ is more than $\frac{1}{2}$.

NOTES

NOTES